of view, we must suppose that the direction in which the characteristic ray travels depends upon neighboring atoms as well as upon the angle of incidence of the primary beam.

The relative reflecting power of different planes may be determined by reflecting from them the *same* portion of the continuous spectrum, or different portions having known relative intensities. The use of the continuous spectrum for this purpose instead of the line spectrum has the advantage of flexibility. One can employ short, penetrating X-rays, and with these investigate dense crystals of high absorbing power.

[1] *Phys. Rev., Ithaca,* Aug. 1915, p. 166.

[2] *J. Optical Soc.,* May 1921, p. 386.

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*ACOUSTIC TOPOGRAPHY VARYING WITH THE POSITION OF THE ORGAN PIPE**

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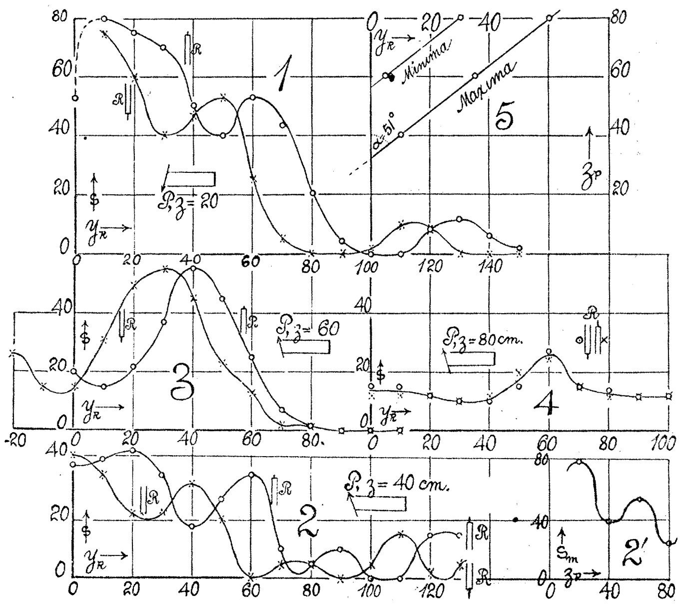
1. *Introductory.*—The coördinate axes have the x-y plane on the large table, the origin near one corner. The survey is made between walls at $y = 174$ cm. and $y = -130$ cm.; and the pipe is displaced both in the y and z (upward) direction. In the graphs, the letters P, R, T, T', used either as subscripts or directly, will refer to the pipe, the resonator, the edges of the table. Throughout s is the fringe deflection of the U-tube interferometer, proportional to the acoustic pressure or to the nodal density evoked at a given place ($x = 0, y, z$), by the sounding pipe P.

The latter was a closed brass f'' pipe, blown by the pipe blower elsewhere described, rigidly attached. In graphs 1–3, this is sketched in under P and an arrow shows the direction of the wind current. The closed cylindrical pinhole resonator R, lying with its axis parallel to y, in the yx plane (table) and moving end-on along y, is also suggestively indicated, showing the resonator azimuth for which the usually paired graphs apply. In the earlier work (figures 1–3) the *middle* of the resonator was used for the location of points in y. An inversion of the resonator thus gave the two curves for each case and they were obtained consecutively. If the mouth of R is used to determine the coördinate y as in figures 4, 6 to 10, but a single curve is obtained, for all azimuths of R. The same is true for a resonator placed with its mouth at the mean y_R of a given ordinate, s, in case of figures 1–3.

2. *Pipe Elevated above the Origin on the Table.*—In figures 1–4 the organ pipe, with its mouth vertically above the origin, was raised succes-

sively in steps of 20 cm. from $z = 20$ cm. to $z = 80$. For each pipe level the distribution of nodal intensity along the y axis was explored by moving R along it (with observations taken every $\Delta y_R = 10$ cm. apart). In figure 1, $z_p = 20$ cm., there is probably some distortion at the origin, owing to the wind from the near pipe, otherwise the first crest would be in negative y. From the glancing incidences, this graph has less detail than figure 2 ($z_p = 40$ cm.), which preserves the same general character but is lower in s and shrunk in the y direction. In figure 3, $z_p = 60$ cm., the topography is totally changed. There is now a trough



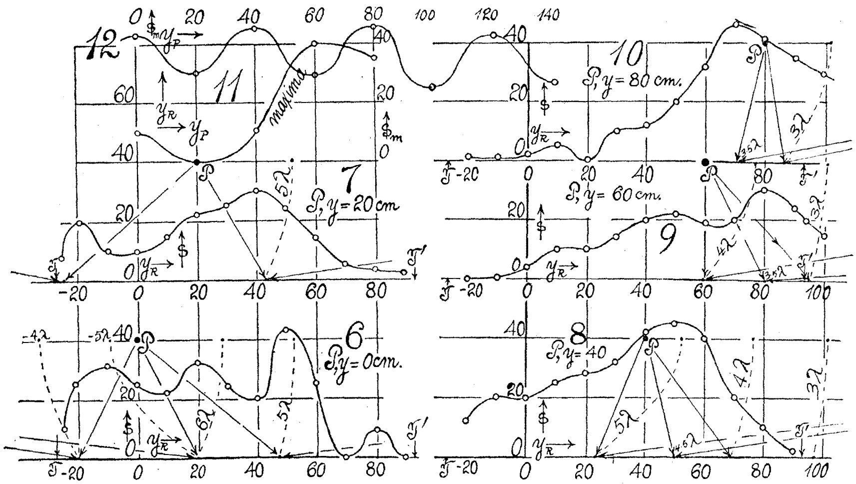
near the origin and (on compounding) but one marked crest. Figure 4, $z_p = 80$ cm., is redeveloping a crest at the origin, but otherwise preserves the former outlines. In this figure and the following, the mouth of the resonator was used to define y_R so that the two curves are practically coincident.

If the highest crests correspond to like path differences and therefore belong together, their position y_R, z_p in figure 5 is seen to progress roughly along a straight line $y = 1.25z - 40$, the slope dy/dz corresponding to an angle of incidence of $\alpha = 51°$. The minima determine parallel lines, the vertical distance apart of consecutive lines being here $\Delta z_p = 24$ cm. or about half a wave-length.

Furthermore, the graphs, as a whole, are alternately high and low in intensity, s. If the intensity s_m of the highest crest be expressed in terms of the pipe elevation z_p, the resulting graph, figure 2′ (scale 1/2), presents

the same strongly marked harmonic character of the reflection graphs of the preceding paper. Cf. § 3.

3. *Pipe Displaced in a Given Level along y.*—Figures 6–10 are constructed on the same plan as the preceding, the ordinates s, denoting fringe deflections or the nodal intensity, encountered by the mouth of the resonator at y_R. If the ordinates be also taken as z_p, the pipe position, P, etc., may also be indicated on the graphs. T, T' show the edges of the table. In figure 6, where the pipe is 40 cm. above the origin, the graph is highly detailed with crests mostly 30 cm. apart. The hyperbolas



for reflections from the wall at y = 174 cm. and beyond T' have been sketched in at 5λ and 6λ of path difference, together with the corresponding acoustic rays. So also those for reflection from the wall at y = − 130 cm. (beyond T). The last agrees badly with the crest observed at about −10 cm. The alluring possibility of an intersection of the positive and negative hyperbolas resulting in very high crests does not work out. In figure 7 also the positive hyperbola 5λ fairly corresponds to the observed crest, but not the negative one. In figures 8, 9, 10 the positive hyperbolas 3λ, 4λ, 5λ, also, fail of coincidence with the observed crests. These in fact lie nearly between them, or half a wave-length of path difference beyond. Thus it seems improbable that the maxima and minima can be explained as resulting from the reflection at these walls, while all other walls and the ceiling are far more distant.

In figures 6–10 one is often in doubt as to which of the crests belong together. Crests seem rather to separate and recombine. Taking the highest in the successive curves, figure 11 shows that the relation of pipe and resonator positions for maxima is here not simple as in figure 5. In fact the pipe approaches the high crest and finally overtakes it (figure 10).

There is no symmetry of graph at the middle of the table ($y = 33$) and the two edges T, T' show no correspondence. The graphs may actually be traced into free air beyond the table, as indicated in the prolongation of figure 12. Finally if the intensity s_m of the supposedly chief maxima be laid off in terms of the pipe position an harmonic figure 12 of relatively constant amplitude appears in reasonable contrast with the curve of diminishing amplitude in figure $2'$, for the raised pipe. In both cases, the steps of 20 cm. between pipe positions are too large for sharp discriminations; but it is noticeable that the distance between crests in figures $2'$, 12 and the earlier reflection figures is about $\Delta y = 40$ cm. The prevailing Δy in the single graphs 2, 6, etc., is about 30 cm. In figure 12 the points beyond $y_p = 80$ cm. were obtained in similar investigations, here omitted.

4. *Summary.*—The above work as a whole has shown that when the reflecting plate is relatively near, to the pipe and resonator, the position of maxima and minima may be predicted as a case of ordinary interference and the wave-length computed satisfactorily. The distribution of intensity among the crests and troughs remains harmonic and has not been foreseen.

If the reflecting surfaces are relatively remote the positions of crests and troughs cannot, as a rule, be found by the same method satisfactorily. In certain instances, there seemed to be an approximate fit; but as a whole the attempt was unsuccessful and the distribution of nodal intensities equally puzzling. Acoustic topography is not symmetrical to the pipe. It appeared, however, that, when the organ pipe, definitely placed, is sounded, the occurrence of nodal surfaces of a fixed position in free air is demonstrable everywhere. Distributions between walls are consistent and differ from distributions between wall and door. It is not improbable, moreover, that such surfaces are regularly grouped and may for reasonable distances be approximately parallel. If then they are intersected obliquely, for instance by the plane of the table, and if α is the mean angle between the latter and the nodal surface, the distance between crests on the table should be $\Delta y = \lambda/(2 \sin \alpha)$. Hence this intercept, Δy, may have any value depending upon the general shape of the room. If the prevailing value $\Delta y = 30$ cm. be taken, $\sin \alpha = 24/30$ or $\alpha = 53°$ roughly. This angle between surfaces is also the angle of incidence of the rays, and has been encountered more or less closely in other instances (fig. 5). The distribution of intensities would then also depend on the fixed pipe position in relation to the given room, regarded integrally as an enveloping reflector. Such an explanation is plausible and flexible, no doubt; but it has the great disadvantage that none of the results can be reproduced by computation. The conviction that some other explanation may be found is not removed and the answer probably lurks in such curves as figures $2'$ and 12. This is particularly so, because the pin-

hole resonator is practically unresponsive to antinodes and to wave trains except within ranges from the pipe which are small as compared with the dimensions of the room as a whole.

* Advance note from a Report to the Carnegie Institution of Washington, D. C.

ON STEERING AN AUTOMOBILE AROUND A CORNER

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The question of so arranging the steering gear of an automobile that the front wheels shall skid as little as possible owing to axes not intersecting has been dealt with by A. L. Candy (*American Mathematical Monthly*, June 1920) and leads to a geometrical discussion of a high degree of complexity. The present somewhat trifling paper has no such ambitious purpose, but was suggested by a question asked on the witness-stand as to how much an automobile must turn out in order to pass another. It was necessary to make some assumption, and this led to the question of steering in general, and the principles that should govern it. We shall consider only the question of passing from a straight course to another at right angles therewith at constant velocity, and inquire how we must steer in order not to skid.

The layman ignorant of geometry and dynamics will suppose that we describe a circle, but this is impossible, not only because it is impossible to put the helm over suddenly so as to change from zero to a finite curvature, but also because the sudden change in the centrifugal force from zero to a finite quantity would produce a wholly intolerable shock. Inasmuch as the question of transition railroad curves has received so much attention, it is thought that the treatment of this question for an automobile may have some interest to engineers. We shall for the present neglect the fact that the rear wheels do not track with the forward ones, taking that up later as a correction. Thus we shall examine the curve described by a machine of zero wheel-base, like a wheelbarrow, measuring wheel or monocycle. The condition is evidently that the curvature shall begin at zero, increase to a maximum until the tangent has turned through an angle of 45 degrees, and then return by a symmetical process.

The centrifugal force is equal to mv^2/ρ, where m is the mass, ρ the radius of curvature, v the velocity, and if there is to be no skidding this must be less than $mg\mu$ where μ is the coefficient of friction, consequently we must have at all times

$$v^2/\rho < \mu g.$$